7.3.1 DEFINITION

The slope of a line is a number that measures its "steepness", usually denoted by the letter m. It is the change in y for a unit change in x along the line.

Given two points (x_1, y_1) and (x_2, y_2), the formula for the slope of the straight line going through these two points is:

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

where the subscripts merely indicate that you have a "first" point (whose coordinates are subscripted with a "1") and a "second" point (whose coordinates are subscripted with a "2"); that is, the subscripts indicate nothing more than the fact that you have two points to work with. Note that the point you pick as the "first" one is irrelevant; if you pick the other point to be "first", then you get the same value for the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLES

1. Let's say that points (15, 8) and (10, 7) are on a straight line. What is the slope of this line?
   a) **Step One:** Identify two points on the line.

   In this example we are given two points, (15, 8) and (10, 7), on a straight line.

   b) **Step Two:** Select one to be ($x1$, $y1$) and the other to be ($x2$, $y2$).

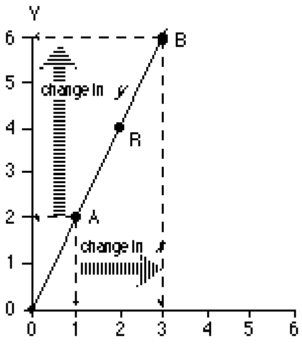
   It doesn't matter which we choose, so let's take (15, 8) to be ($x2$, $y2$). Let's take the point (10, 7) to be the point ($x1$, $y1$).

   c) **Step Three:** Use the equation to calculate slope.

   Once we've completed step 2, we are ready to calculate the slope using the equation for a slope:

   Slope m = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{8-7}{15-10} = \frac{1}{5}$

2. What is the slope of the line given in the graph?



a) **Step One:** Identify two points on the line.
Let's calculate the slope of the line in the graph above using the points A (1, 2) and B (3, 6).

b) **Step Two:** Select one to be (*x*1, *y*1) and the other to be (*x*2, *y*2).
Let's take A (1, 2) to be (*x*1, *y*1). Let's take the point B (3, 6) to be the point (*x*2, *y*2).

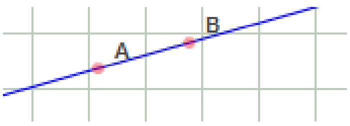
c) **Step Three:** Use the equation to calculate slope.

$$slope = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(6-2)}{(3-1)} = \frac{4}{2} = 2$$

SLOPE DIRECTION

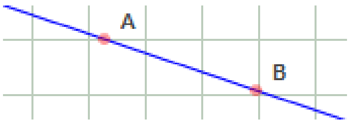
The slope of a line can be positive, negative, zero or undefined.

1. POSITIVE SLOPE



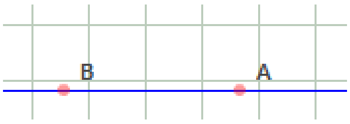
Here, y *increases* as x increases, so the line slopes upwards to the right. The slope will be a positive number. The line on the right has a slope of about +0.3, it goes *up* about 0.3 for every step of 1 along the x-axis.

2. NEGATIVE SLOPE



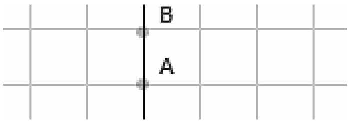
Here, y *decreases* as x increases, so the line slopes downwards to the right. The slope will be a negative number. The line on the right has a slope of about -0.3, it goes *down* about 0.3 for every step of 1 along the x-axis.

3. ZERO SLOPE



Here, y *does not change* as x increases, so the line in exactly horizontal. The slope of any horizontal line is always zero. The line on the right goes neither up nor down as x increases, so its slope is zero.

4. UNDEFINED SLOPE



When the line is exactly vertical, it does not have a defined slope. The two x coordinates are the same, so the difference is zero. The slope calculation is then something like

$$\text{slope} = \frac{21}{0}$$

When you divide anything by zero the result has no meaning. The line above is exactly vertical, so it has no defined slope. We say "the slope of the line AB is undefined".

A vertical line has an equation of the form x = a, where a is the x-intercept. For more on this see [Slope of a vertical line].

SELF-ASSESSMENT ACTIVITY

1. Calculate the gradient of the line passing through (- 6, -5) and (4, 1)

2. Calculate the gradient of the line through (5, 1) and (5, 6)

3. Calculate the slope of a line passing through (3, 2) and (4, 2)