7.4.4 THE EQUATION OF A STRAIGHT LINE WITH GIVEN GRADIENT, PASSING THROUGH A GIVEN POINT

Suppose that we want to find the equation of a line which has a gradient of $\frac{1}{3}$ and passes through the point (1, 2). Here, whilst we know the gradient, we do not know the value of the y-intercept c.

We start with the general equation of a straight line y = mx + c.

We know the gradient is 1/3 and so we can substitute this value for m straightaway. This gives

y = $\frac{1}{3}$ x + c.

We now use the fact that the line passes through (1, 2). This means that when x =1, y must be 2. Substituting these values we find

$2 = \frac{1}{3} \times 1 + c$

So that $c = 2 - \frac{1}{3} = \frac{5}{3}$

So the equation of the line is $y = \frac{1}{3}x + \frac{5}{3}$

We can work out a general formula for problems of this type by using the same method. We

shall take a general line with gradient m, passing through the fixed point A(x_1,y_1).

We start with the general equation of a straight line y = mx + c.

We now use the fact that the line passes through A(x_1,y_1). This means that when x = x_1, y must be y_1. Substituting these values we find:

$y_1 = mx_1 + c$

so that $y_1 - mx_1$

So the equation of the line becomes $y = mx + y_1 - mx_1$

We can write alternatively $y - y_1 = m(x - x_1)$

This then represents a straight line with gradient m, passing through the point (x_1,y_1). So this general form is useful if you know the gradient and one point on the line.

The equation of a straight line with gradient m, passing through the point (x_1 ,y_1), is

$y — y_1 = m(x - x_1)$.

For example, suppose we know that a line has gradient -2 and passes through the point (-3, 2). We can use the formula $y - y_1 = m(x - x_1)$ and substitute in the values straight away:

$y - 2 = -2(x - (-3)) = -2(x + 3) = -2x - 6$

$y = -2x - 4$

SELF-ASSESSMENT ACTIVITY

1. A line through (2, 6) has a slope $\frac{2}{3}$. Find its equation and graph it

2. Find the equation of the lines described below (give the equation in the form y = mx + c):

   (a) gradient 3, passing through (1,4);

   (b) gradient -2, passing through (2,0);

   (c) gradient 2/5, passing through (5,-1);

   (d) gradient 0, passing (-1,2);

   (e) gradient -1, passing through (1, -1).