7.4.5 THE EQUATION OF A STRAIGHT LINE THROUGH TWO GIVEN POINTS

What should we do if we want to find the equation of a straight line which passes through the

two points (-1, 2) and (2,4)?

Here we don't know the gradient of the line, so it seems as though we cannot use any of the formula we have found so far. But we do know two points on the line, and so we can use them to work out the gradient. We just use the formula $\boldsymbol{m} = \frac{\boldsymbol{y2 - y1}}{\boldsymbol{x2 - x1}}$. We get

$$m = \frac{4-2}{2-(-1)} = \frac{2}{3}$$

So the gradient of the line is $\frac{2}{3}$. And we know two points on the line, so we can use one of them in the formula $y - y_1 = m(x - x_1)$. If we take the point (2,4) we get

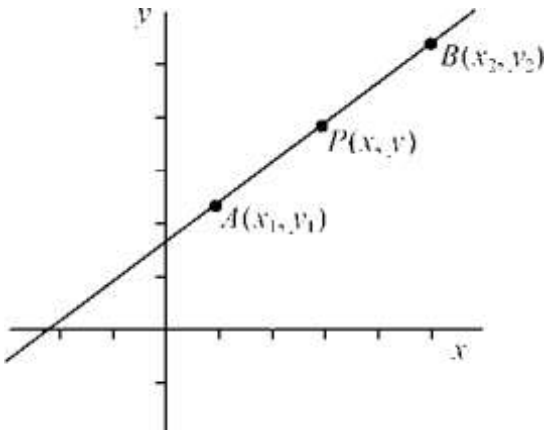
$\boldsymbol{y - 4} = \frac{2}{3}x - 2$

$3y - 12 = 2x - 4$

$3y = 2x + 8$

$\mathbf{y} = \frac{2}{3}x + \frac{8}{3}$

As before, it will be useful to find a general formula that can be used for examples of this kind. So suppose the general line passes through two points $A(x_1, y_1)$ and $B(x_2, y_2)$. We shall let a general point on the line be P(x,y).



Now we know that the gradient of AP must be the same as the gradient of AB, as all three points

are on the same line. But the gradient of AP is

$$m_{AP} = \frac{y - y_1}{x - x_1}$$

whereas the gradient of AB is $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

Then $m_{AP} = m_{AB}$, so we must have $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ and after rearranging we get

$$\boldsymbol{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$

This is the general equation of a straight line passing through the two points (x_1, y_1) and (x_2, y_2)

Now we can use this formula for an example. Suppose that we want to find the equation of the straight line which passes through the two points (1, —2) and (—3,0). We just substitute into the formula, and rearrange.

$y - (-2) = \frac{0-(-2)}{-3-1}(x - 1)$

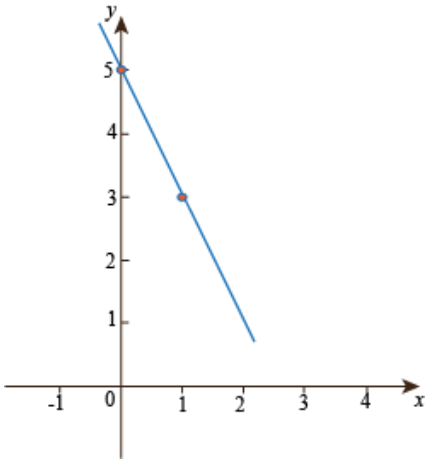
$\boldsymbol{y + 2 = \frac{2}{-4}(x - 1)}$

$\boldsymbol{y = -\frac{1}{2}x + \frac{1}{2} - 2 \; or \; y = -\frac{1}{2}x - \frac{3}{2}}$

SELF-ASSESSMENT ACTIVITY

1. Find the equation of the lines described below (give the equation in the form y = mx + c):
   a) passing through (4,6) and (8, 26),
   b) passing through (1,1) and (4, -8),
   c) passing through (3,4) and (5,4),
   d) passing through (0, 2) and (4,0),
   e) passing through (-2, 3) and (2, -5).

3. Find the equation of the line graphed below.



4. Find the equation of the line graphed below.